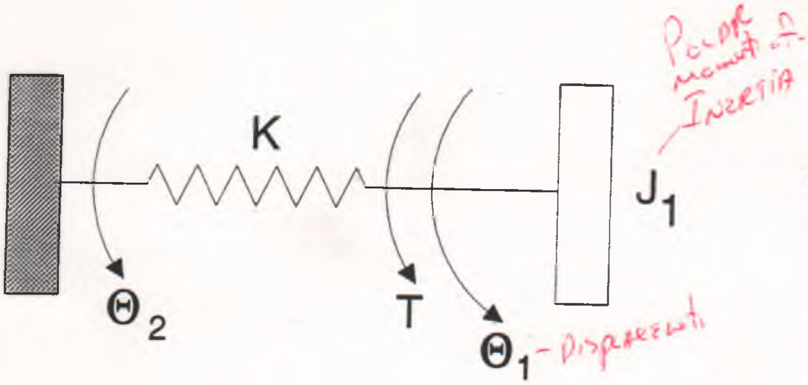
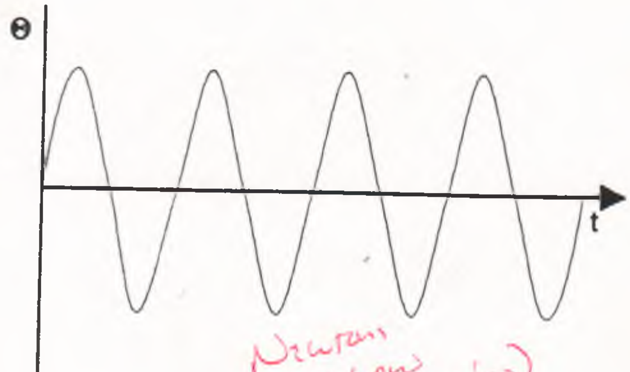
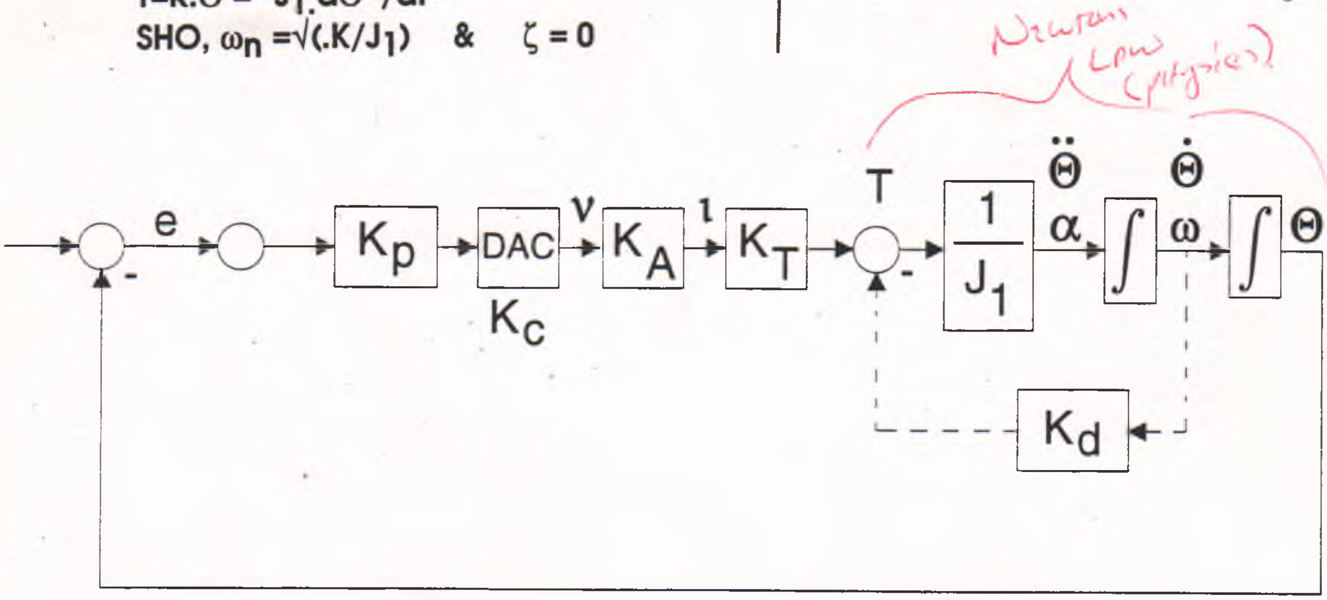
Proportional Control





$T=K.\Theta = -J_1.d\Theta^2/dt^2$

SHO, $\omega_n = \sqrt{(.K/J_1)}$ & $\zeta = 0$



Assuming small friction ($K_d \approx 0$),

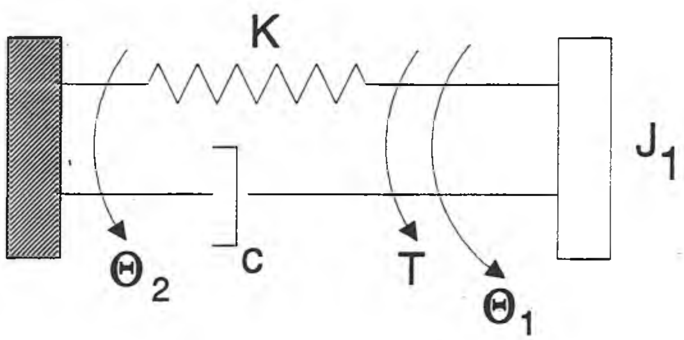
$$T(t)= K_p.K_C.K_A.K_T.e(t) = - K_p.K_C.K_A.K_T.\Theta\ (t) = J_1.d\Theta^2/dt^2$$

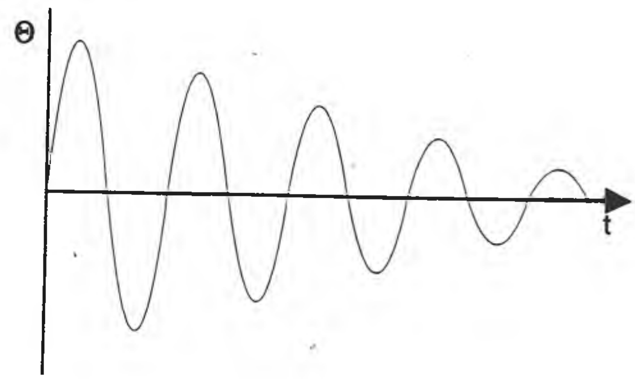
This is an undamped SHO.

$$\omega_n = \sqrt{((K_p.K_C.K_A.K_T.)/J_1)}\ \&\qquad \zeta = 0$$

- Thus the proportional gain $K_p \rightarrow$ spring stiffness
- Higher $K_p \rightarrow$ higher Stiffness

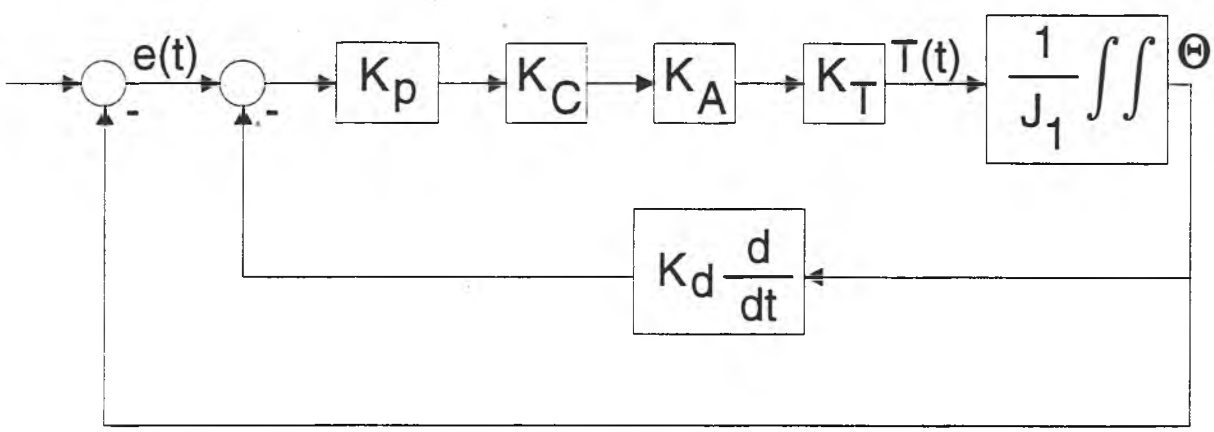
Derivative Control





$T = K.\Theta + c.\, d\Theta/dt = -J_1 . d\Theta^2/dt^2$

damped SHO, $\omega_n = \sqrt{(.K/J_1)}$ & $\zeta = (c/2\sqrt{(k.J_1)}$

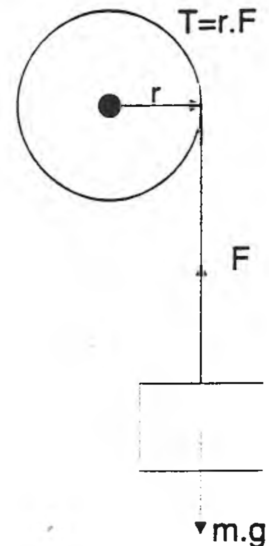


$T(t) = -K_p.K_C.K_A.K_T.\Theta\ (t) - K_p.K_C.K_A.K_T.K_d.\ d\Theta/dt = J_1.d\Theta^2/dt^2$

This is a <u>damped</u> SHO,

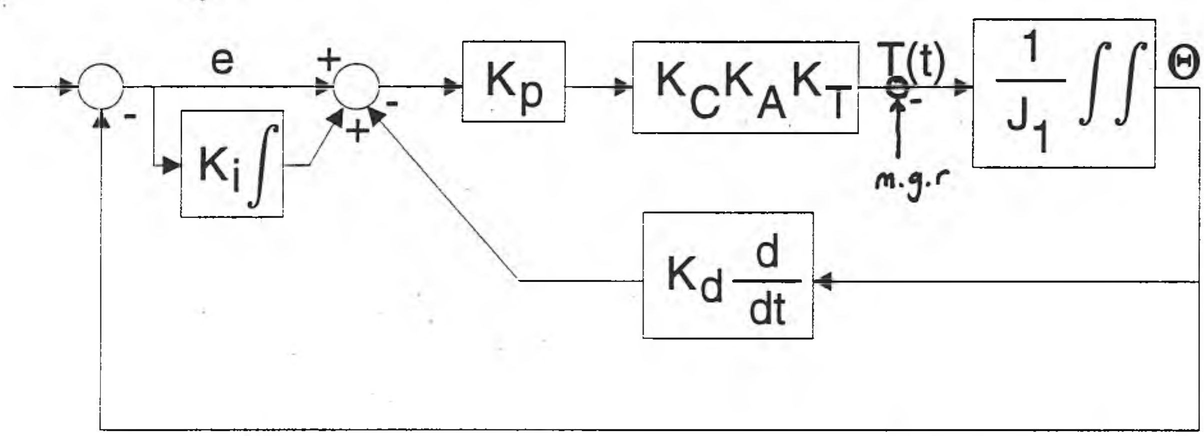
$\omega_n = \sqrt{((K_p.K_C.K_A.K_T.)/J_1)}$ & $\zeta = (K_d/2\sqrt{(J_1/\ K_p.K_A.K_T.K_C)})$

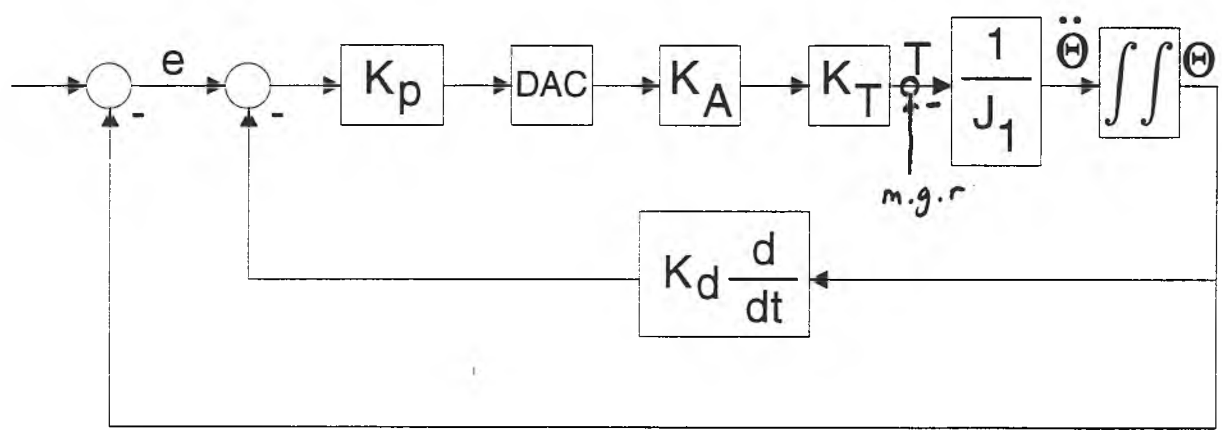
Integral Term



Without Integral:

$T(t) = K_p.K_C.K_A.K_T.\underline{e(t)} = m.g.r$





With Integral

$T(t) = (K_p.K_C.K_A.K_T).(K_i\int e(t)dt + e(t)) = m.g.r$

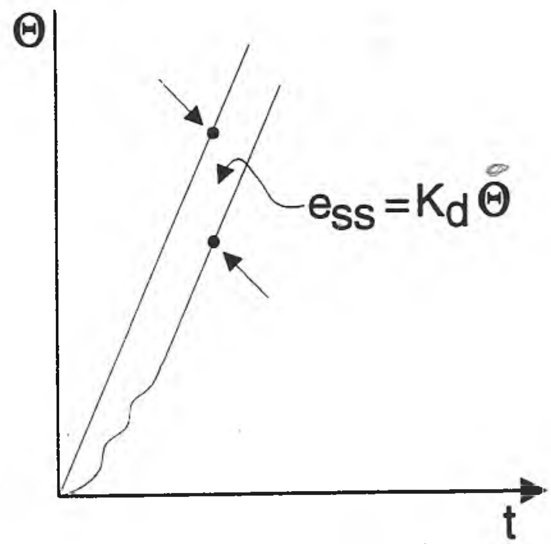
therefore as $t \rightarrow \infty$, $e(t) \rightarrow 0$

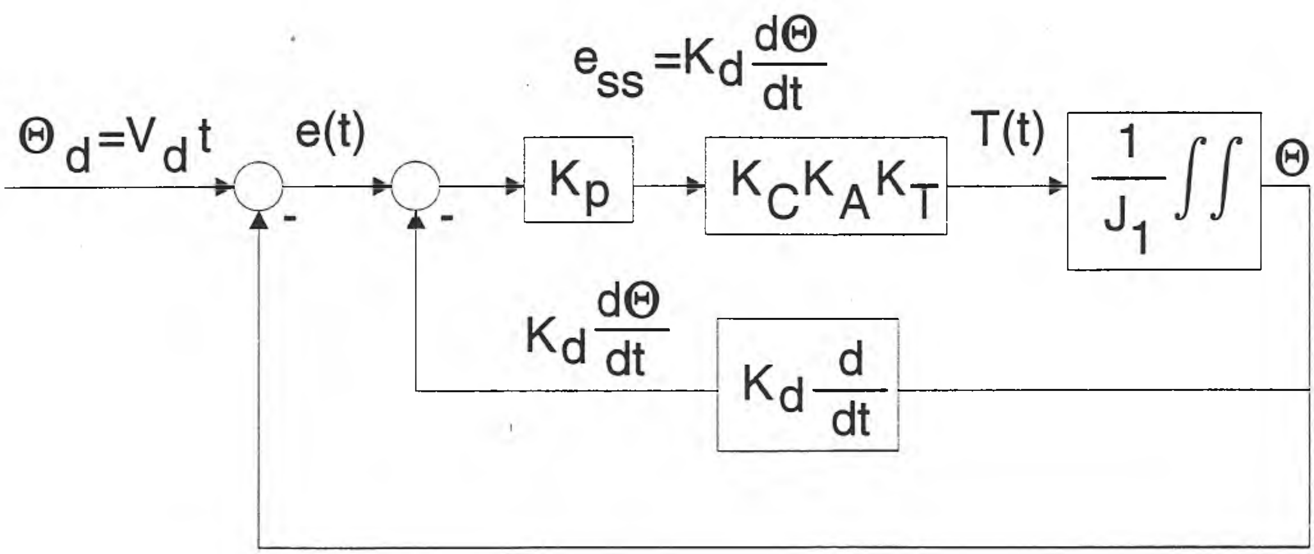
Errors at Steady State Due Constant Speed Trajectory Tracking

Ramp Input: $\Theta_d = V_d.t$, $d\Theta/dt = V_d$, $d\Theta^2/dt^2 = 0$

Results (assuming friction ≈0):

$T_{ss} \propto e_{ss} - K_d.d\Theta/dt = 0$

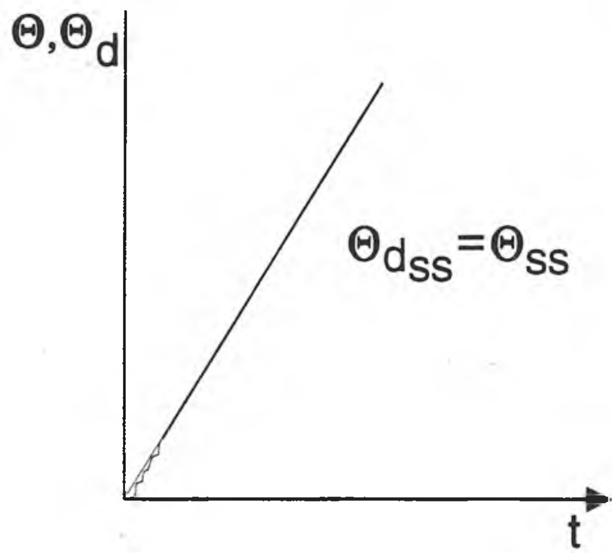


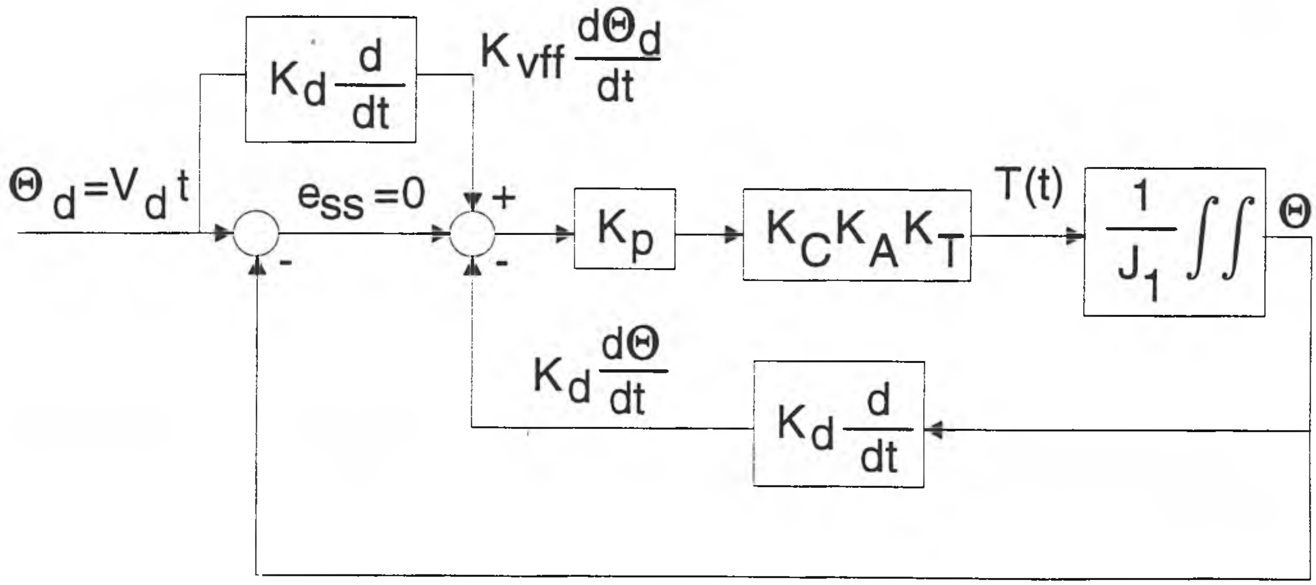


Purpose of Velocity Feedforward

Selecting $K_{vff} = K_d$ $\rightarrow$ $e_{ss} = 0$

No *steady State* error due to constant velocity tracking.





Purpose of Acceleration Feedforward

In general, the trajectories contain higher order time functions:

e.g. : Constant jerk trajectory $\Theta_d(t) = c_0 + c_1.t + c_2.t^2$

By choosing,

$$K_{aff} = J_1/(K_p.K_C.K_A.K_T) \rightarrow \quad e(t) = 0$$

This results in no tracking error for the *Ideal* system.

Since,

$$T(t) = (J_1/(K_p.K_C.K_A.K_T)).d^2\Theta_d/dt.^2(K_p.K_C.K_A.K_T) = J_1.d^2\Theta_d/dt.^2$$

$\rightarrow d^2\Theta_d/dt.^2 = d^2\Theta/dt.^2 \rightarrow \quad \Theta_d(t) = \Theta(t) \rightarrow$ No tracking errors